Fermionic dispersion relations in ultradegenerate relativistic plasmas beyond leading logarithmic order

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We determine the dispersion relations of fermionic quasiparticles in ultradegenerate plasmas by a complete evaluation of the on-shell hard-dense-loop-resummed one-loop fermion self energy for momenta of the order of the Fermi momentum and above. In the case of zero temperature, we calculate the nonanalytic terms in the vicinity of the Fermi surface beyond the known logarithmic approximation, which turn out to involve fractional higher powers in the energy variable. For nonzero temperature (but much smaller than the chemical potential), we obtain the analogous expansion in closed form, which is then analytic but involves polylogarithms. These expansions are compared with a full numerical evaluation of the resulting group velocities and damping coefficients.

I. INTRODUCTION

Unscreened magnetostatic interactions in a degenerate Fermi gas lead to dramatic changes of the fermionic dispersion relation in the vicinity of the Fermi surface. At strictly zero temperature, there is a logarithmic singularity in the inverse group velocity, which leads to a breakdown of the Fermi liquid picture. This effect has been discovered in the context of a nonrelativistic degenerate electron gas by Holstein, Norton, and Pincus [1] over thirty years ago, who found that it gives rise to an anomalous $T \ln T^{-1}$ behavior of the low-temperature specific heat (see also [2, 3, 4]).

In deconfined degenerate quark matter the same effect is caused by unscreened chromomagnetic fields, since the nonperturbative magnetic screening is parametrically of the order g^2T and thus vanishes in the low-temperature limit [5]. Although chromomagnetic screening may arise in the form of a Meissner effect in a color superconducting phase, the appearance of logarithmic terms in the quark self-energy at (resummed) one-loop order is also of importance in the case of color superconductivity, since it leads to a a significant reduction of the magnitude of the superconductivity gap in a weak-coupling analysis [6, 7].

In the normal phase of degenerate quark matter, non-Fermi-liquid behavior leads to anomalous specific heat which because of the greater number of gauge bosons and the stronger coupling is comparatively large. Moreover, as has been shown in Ref. [8], the relevant logarithms are stable in the sense that they do not exponentiate into power-law behavior when higher loop orders are included as was previously assumed [9, 10]. However, an actual numerical evaluation requires to go beyond the leading logarithmic order calculations performed in the condensed-matter context [1, 2, 3, 4]. This has been recently achieved for the low-temperature specific heat, where the scale of the leading temperature logarithm as well as subleading fractional powers of temperature were determined in Ref. [11, 12], in a calculation which circumvented a complete evaluation of the fermion propagator.

Non-Fermi-liquids effects have also been shown recently to significantly enhance the neutrino emission rate from normal quark matter [13]. However, this calculation involves the fermionic dispersion relations which have so far been known only to leading logarithmic accuracy.

In this paper we shall close this gap and present a complete evaluation of the on-shell hard-dense-loop-resummed one-loop fermion self energy for momenta of the order of the Fermi momentum and above, both at zero and at small temperatures.

II. FERMION SELF ENERGY ON THE LIGHT CONE

The fermion self energy is defined through

$$S^{-1}(P) = S_0^{-1}(P) + \Sigma(P), \tag{1}$$

where $S_0(P) = -(\not{P})^{-1}$ is the free fermion propagator, and $P^\mu = (p^0, \vec{p})$ with $p^0 = i\omega_n + \mu$, $\omega_n = (2n+1)\pi T$ in the imaginary time formalism, and $P^\mu = (E, \vec{p})$ after analytic continuation to Minkowski space. Without loss of generality we shall assume that $\mu > 0$.

With the energy projection operators $\Lambda^\pm_{\mathbf{p}} = \frac{1}{2}(1 \pm \gamma_0 \gamma^i \hat{p}^i)$ we decompose $\Sigma(P)$ in the quasiparticle and antiquasiparticle self energy,

$$\Sigma(P) = \gamma_0 \Lambda^+_{\mathbf{p}} \Sigma_+(P) - \gamma_0 \Lambda^-_{\mathbf{p}} \Sigma_-(P), \tag{2}$$

and

$$\gamma_0 S^{-1} = \Delta_+^{-1} \Lambda^+_{\mathbf{p}} + \Delta_-^{-1} \Lambda^-_{\mathbf{p}} \tag{3}$$

so that $\Delta_\pm^{-1} = -[p^0 \mp (|\mathbf{p}| + \Sigma_\pm)]$.

The one-loop fermion self energy is given by

$$\Sigma(P) = -g^2 C_f T \sum_\omega \int \frac{d^3q}{(2\pi)^3} \gamma^\mu S_0(P-Q) \gamma^\nu \Delta_{\mu\nu}(Q), \tag{4}$$

where $\Delta_{\mu\nu}$ is the gauge boson propagator. Following [20] we introduce an intermediate scale q^*, such that $m \ll q^* \ll \mu$, and we divide the q integration into a soft part ($q < q^*$) and a hard part ($q > q^*$),

$$\Sigma_+ = \Sigma_+^{(\mathrm{s})} + \Sigma_+^{(\mathrm{h})}. \tag{5}$$

For the hard part we can use the free gluon propagator, whereas for the soft part we have to use a resummed gluon propagator, see below.

The hard contribution to Σ_+ on the light cone is most easily computed in covariant Feynman gauge, with the result

$$\Sigma_+^{(\mathrm{h})} = \frac{M_\infty^2}{2p}, \tag{6}$$

with $M_\infty^2 = g^2 C_f \mu^2/(4\pi^2)$. Here q^* enters only as a correction proportional to q^*/μ, so that we can send q^* to zero. Correspondingly we expect that in the soft contribution we should be able to send q^* to infinity without encountering divergences, as will indeed be the case, but only after all soft contributions are added together.

The leading and next-to-leading contributions to the soft part of the fermion self energy for momenta of the order of μ or larger are obtained by a one-loop diagram where the fermion propagator is a bare propagator, but the gauge boson propagator is dressed by so-called hard-dense-loop (HDL) [14, 15, 16, 17] self energies, so that the transverse and longitudinal parts of the gluon propagator are given by [18]

$$\Delta_T(q_0, q) = \frac{-1}{q_0^2 - q^2 - m^2 \frac{q_0^2}{q^2}\left[1 + \frac{q^2 - q_0^2}{2qq_0} \log\left(\frac{q_0+q}{q_0-q}\right)\right]}, \tag{7}$$

$$\Delta_L(q_0, q) = \frac{-1}{q^2 + 2m^2\left[1 - \frac{q_0}{2q}\log\left(\frac{q_0+q}{q_0-q}\right)\right]}. \tag{8}$$

We shall consider zero or small temperature $T \ll \mu$, for which the mass parameter in the HDL propagator is given by

$$m^2 = \frac{N_f g^2 \mu^2}{4\pi^2}, \tag{9}$$

which is the asymptotic mass of the transverse modes, related to the Debye screening mass m_D by $m^2 = m_D^2/2$.

On the light cone one finds the gauge-independent expression [19]

$$\begin{aligned}\Sigma_\pm^{(\mathrm{s})}(E) = &-\frac{g^2 C_f}{8\pi^2}\int_0^{q^*} dq\, q^2 \int_{-1}^{1} dt \int_{-\infty}^{\infty} dk_0\, [\delta(k_0-k)-\delta(k_0+k)] \\ &\times\Bigg\{ 2\left(\pm\mathrm{sgn}(k_0) - \hat{\mathbf{p}}\cdot\hat{\mathbf{q}}\,\hat{\mathbf{k}}\cdot\hat{\mathbf{q}}\right)\int_{-\infty}^{\infty}\frac{dq_0}{2\pi}\rho_T(q_0,q)\frac{1+n_b(q_0)-n_f(k_0-\mu)}{k_0+q_0\mp|E|-i\epsilon} \\ &+\left(\pm\mathrm{sgn}(k_0)+\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}\right)\Bigg[\int_{-\infty}^{\infty}\frac{dq_0}{2\pi}\rho_L(q_0,q)\frac{1+n_b(q_0)-n_f(k_0-\mu)}{k_0+q_0\mp|E|-i\epsilon} \\ &-\frac{1}{q^2}\left(\frac{1}{2}-n_f(k_0-\mu)\right)\Bigg]\Bigg\},\end{aligned} \tag{10}$$

where $\mathbf{k} = \mathbf{p} - \mathbf{q}$ and $E = \pm p$. The distribution functions are given by $n_b(q_0) = 1/(e^{q_0/T} - 1)$ and $n_f(k_0 - \mu) = 1/[e^{(k_0-\mu)/T} + 1]$. ρ_T and ρ_L are the spectral densities of tranverse and longitudinal gauge bosons, respectively,

$$\rho_{T,L}(q_0, q) = 2\mathrm{Im}\Delta_{T,L}(q_0 + i\epsilon, q). \tag{11}$$

We may use $q \ll |E|, k$ because of $q < q^*$ and $|E| \gtrsim \mu$. Depending on the sign of E, we can drop the term $\delta(k_0 + k)$ or the term $\delta(k_0 - k)$ in Eq. (10), since its contribution is suppressed with $\sim q/E$ compared to the remaining contribution. Then we find for the soft contribution to the real part of Σ_+

$$\begin{aligned}\mathrm{Re}\,\Sigma_+^{(\mathrm{s})} = &-\frac{g^2 C_f}{8\pi^2}\int_0^{q^*} dq\, q^2\int_{-1}^{1} dt \Bigg[\int_{-\infty}^{\infty}\frac{dq_0}{\pi}\left[(1-t^2)\rho_T(q_0,q)+\rho_L(q_0,q)\right] \\ &\times\mathcal{P}\frac{1+n_b(q_0)-n_f(E-\mu-qt)}{q_0-qt} - \frac{1}{q^2}\left(1-2n_f(E-\mu-qt)\right)\Bigg].\end{aligned} \tag{12}$$

This quantity vanishes for $E = \mu$ by symmetric integration. After performing the q_0-integration we therefore have

$$\begin{aligned}\mathrm{Re}\,\Sigma_+^{(\mathrm{s})} = &\frac{g^2 C_f}{4\pi^2}\int_0^{q^*} dq\, q^2 \int_{-1}^{1} dt\,(n_f(E-\mu-qt)-n_f(-qt)) \\ &\times\left[(1-t^2)\mathrm{Re}\,\Delta_T(qt,q)+\mathrm{Re}\,\Delta_L(qt,q)\right].\end{aligned} \tag{13}$$

For $\mathrm{Im}\,\Sigma_+$ (which receives no hard contribution) we find in an analogous way

$$\begin{aligned}\mathrm{Im}\,\Sigma_+ = &-\frac{g^2 C_f}{8\pi^2}\int_0^{q^*} dq\, q^2\int_{-1}^{1} dt\left((1-t^2)\rho_T(qt,q)+\rho_L(qt,q)\right) \\ &\times\left[1+n_b(qt)-n_f(E-\mu-qt)\right].\end{aligned} \tag{14}$$

The antiquasiparticle self energy $\Sigma_-^{(\mathrm{s})}$ is obtained by inserting negative values of E in the expressions for $\Sigma_+^{(\mathrm{s})}$ and including an overall factor (-1). With $\mu > 0$ we can then replace $n_f(E - \mu - qt)$ by 1.

III. EXPANSION FOR SMALL $|E-\mu|$ AND SMALL T

In this section we will perform an expansion of Σ_+ in the region

$$T \sim |E-\mu| \ll g\mu \ll \mu, \tag{15}$$

where non-Fermi-liquid effects dominate. We will use the expansion parameter $a := T/m$, and we define $\lambda := (E-\mu)/T$. From (15) we have $a \ll 1$ and $\lambda \sim \mathcal{O}(1)$.

In the part with the transverse gluon propagator we substitute $q = ma^{1/3}z$ and $t = a^{2/3}v/z$. After expanding the integrand with respect to a we find for the transverse contribution

$$\begin{aligned}\mathrm{Re}\,\Sigma^{(\mathrm{s})}_{+(T)} &= -\frac{g^2 C_f m a}{\pi^2}\int_{-\frac{q^*}{am}}^{\frac{q^*}{am}} dv \int_{a^{2/3}|v|}^{\frac{q^*}{a^{1/3}m}} dz\, \frac{e^\lambda - 1}{(1+e^v)(1+e^{\lambda-v})} \\ &\times \left[\frac{z^5}{v^2\pi^2+4z^6} + \frac{2v^2 z(v^2\pi^2-4z^6)}{(v^2\pi^2+4z^6)^2}a^{2/3} - \frac{16v^4z^3(3v^2\pi^2-4z^6)}{(v^2\pi^2+4z^6)^3}a^{4/3} + \dots\right].\end{aligned} \tag{16}$$

The z-integrations are straightforward. In the v-integrals we may send the integration limits to $\pm\infty$. Using the formulae

$$\int_{-\infty}^{\infty} dv \frac{e^\lambda-1}{(1+e^v)(1+e^{\lambda-v})}|v|^\alpha = \Gamma(\alpha+1)\left[\mathrm{Li}_{\alpha+1}(-e^{-\lambda}) - \mathrm{Li}_{\alpha+1}(-e^{\lambda})\right] \quad \forall \alpha \geq 0, \tag{17}$$

$$\int_{-\infty}^{\infty} dv \frac{e^\lambda-1}{(1+e^v)(1+e^{\lambda-v})}\log|v| = -\gamma_E\lambda + \frac{\partial}{\partial\alpha}\left(\mathrm{Li}_{\alpha+1}(-e^{-\lambda}) - \mathrm{Li}_{\alpha+1}(-e^{\lambda})\right)\Big|_{\alpha=0}, \tag{18}$$

we find, neglecting terms which are suppressed at least with $(m/q^*)^4$,

$$\begin{aligned}\mathrm{Re}\,\Sigma^{(\mathrm{s})}_{+(T)} &= -g^2 C_f m \\ &\times\Bigg\{\frac{a}{12\pi^2}\left[\lambda\log\left(\frac{2(q^*)^3}{am^3\pi}\right) + \gamma_E\lambda - \frac{\partial}{\partial\alpha}\left(\mathrm{Li}_{\alpha+1}(-e^{-\lambda}) - \mathrm{Li}_{\alpha+1}(-e^{\lambda})\right)\Big|_{\alpha=0}\right] \\ &+\frac{2^{1/3}a^{5/3}}{9\sqrt{3}\pi^{7/3}}\Gamma\left(\tfrac{5}{3}\right)\left(\mathrm{Li}_{5/3}(-e^{-\lambda}) - \mathrm{Li}_{5/3}(-e^{\lambda})\right) \\ &-20\frac{2^{2/3}a^{7/3}}{27\sqrt{3}\pi^{11/3}}\Gamma\left(\tfrac{7}{3}\right)\left(\mathrm{Li}_{7/3}(-e^{-\lambda}) - \mathrm{Li}_{7/3}(-e^{\lambda})\right) \\ &+\frac{8(24-\pi^2)a^3\log a}{27\pi^6}\lambda(\lambda^2+\pi^2) + \mathcal{O}(a^3)\Bigg\}.\end{aligned} \tag{19}$$

In the longitudinal part we substitute $q = mx$ and $t = au/x$. In a similar way as for the transverse part we find

$$\mathrm{Re}\,\Sigma^{(\mathrm{s})}_{+(L)} = -g^2 C_f m\left[\frac{a\lambda}{8\pi^2}\log\left(\frac{2m^2}{(q^*)^2}\right) - \frac{(\pi^2-4)a^3\log a}{96\pi^2}\lambda(\lambda^2+\pi^2) + \mathcal{O}(a^3)\right]. \tag{20}$$

Turning now to $\mathrm{Im}\,\Sigma_+$ we notice that it vanishes at $E=\mu$ only in the case of $T=0$. For finite temperature, however small, there is an IR divergent contribution in the transverse sector [18],

$$\mathrm{Im}\,\Sigma^{(\mathrm{s})}_{+(T)}|_{E=\mu} = -\frac{g^2 C_f T}{4\pi}\ln\frac{m}{\Lambda_{\mathrm{IR}}} \tag{21}$$

where the infrared cutoff may be provided at finite temperature by the nonperturbative magnetic screening mass of QCD. In QED, where no magnetostatic screening is possible, a resummation of these singularities leads to nonexponential damping behavior [21].

After subtraction of the energy independent part we have

$$\mathrm{Im}\,\Sigma_+^{(\mathrm{s})} - \mathrm{Im}\,\Sigma_+^{(\mathrm{s})}|_{E=\mu} = \frac{g^2 C_f}{8\pi^2} \int_0^{q^*} dq\, q^2 \int_{-1}^{1} dt\, (n_f(E-\mu-qt) - n_f(-qt)) \\ \times \left[(1-t^2)\rho_T(qt,q) + \rho_L(qt,q)\right]. \tag{22}$$

Following the steps which led to Eq. (16), we find for the transverse contribution

$$\mathrm{Im}\,\Sigma_{+(T)}^{(\mathrm{s})} - \mathrm{Im}\,\Sigma_{+(T)}^{(\mathrm{s})}|_{E=\mu} = \frac{g^2 C_f m a}{2\pi} \int_{-\frac{q^*}{am}}^{\frac{q^*}{am}} dv \int_{a^{2/3}|v|}^{\frac{q^*}{a^{1/3}m}} dz\, \frac{e^\lambda - 1}{(1+e^v)(1+e^{\lambda - v})} \\ \times \left[-\frac{z^2 v}{v^2\pi^2 + 4z^6} + \frac{16 v^3 z^4}{(v^2\pi^2+4z^6)^2} a^{2/3} + \frac{16 v^5 (v^2\pi^2 - 12 z^6)}{(v^2\pi^2 + 4z^6)^3} a^{4/3} + \dots \right]. \tag{23}$$

Using the formula

$$\int_{-\infty}^{\infty} dv \frac{e^\lambda - 1}{(1+e^v)(1+e^{\lambda-v})} |v|^\alpha \mathrm{sgn}(\alpha) \\ = -\Gamma(\alpha+1)\left[\mathrm{Li}_{\alpha+1}(-e^{-\lambda}) + \mathrm{Li}_{\alpha+1}(-e^{\lambda}) + 2\left(1-2^{-\alpha}\right)\zeta(\alpha+1)\right] \quad \forall \alpha \geq 0 \tag{24}$$

we find in a similar way as above

$$\mathrm{Im}\,\Sigma_{+(T)}^{(\mathrm{s})} - \mathrm{Im}\,\Sigma_{+(T)}^{(\mathrm{s})}|_{E=\mu} = g^2 C_f m \Bigg\{ -\frac{a}{12\pi} \log\cosh\left(\frac{\lambda}{2}\right) \\ -\frac{2^{1/3} a^{5/3}}{9\pi^{7/3}} \Gamma\left(\tfrac{5}{3}\right) \left[\mathrm{Li}_{5/3}(-e^{-\lambda}) + \mathrm{Li}_{5/3}(-e^{\lambda}) + 2\left(1 - 2^{-2/3}\right)\zeta\left(\tfrac{5}{3}\right)\right] \\ -20 \frac{2^{2/3} a^{7/3}}{27\pi^{11/3}} \Gamma\left(\tfrac{7}{3}\right) \left[\mathrm{Li}_{7/3}(-e^{-\lambda}) + \mathrm{Li}_{7/3}(-e^{\lambda}) + 2\left(1 - 2^{-4/3}\right)\zeta\left(\tfrac{7}{3}\right)\right] + \mathcal{O}(a^3) \Bigg\}. \tag{25}$$

For the longitudinal part we obtain

$$\mathrm{Im}\,\Sigma_{+(L)}^{(\mathrm{s})} - \mathrm{Im}\,\Sigma_{+(L)}^{(\mathrm{s})}|_{E=\mu} = -g^2 C_f m \left[\frac{a^2\lambda^2}{64\sqrt{2}} + \mathcal{O}(a^3)\right]. \tag{26}$$

We remark that the determination of the coefficient of the $\mathcal{O}(a^3)$ terms in Σ_+ would require resummation of IR enhanced contributions along the lines of Ref. [12], App. A.

Putting the pieces together, and using the abbreviation $\varepsilon = E - \mu$, we obtain for the real part

$$\mathrm{Re}\,\Sigma_+ = \frac{M_\infty^2}{2E} - g^2 C_f m\, \mathrm{sgn}(\varepsilon) \Bigg\{ \frac{|\varepsilon|}{12\pi^2 m}\left[\log\left(\frac{4\sqrt{2}m}{\pi T f_1(\varepsilon/T)}\right) + 1\right] + \frac{2^{1/3}\sqrt{3}}{45\pi^{7/3}} \left(\frac{T}{m} f_2\left(\frac{\varepsilon}{T}\right)\right)^{5/3} \\ -20 \frac{2^{2/3}\sqrt{3}}{189\pi^{11/3}} \left(\frac{T}{m} f_3\left(\frac{\varepsilon}{T}\right)\right)^{7/3} - \frac{6144 - 256\pi^2 + 36\pi^4 - 9\pi^6}{864\pi^6} \left(\frac{T}{m} f_4\left(\frac{\varepsilon}{T}\right)\right)^3 \log\left(\frac{m}{T}\right) \\ + \mathcal{O}\left(\left(\frac{T}{m}\right)^3\right) \Bigg\}, \tag{27}$$

where

$$f_1(\lambda) \;=\; \exp\left[1-\gamma_E+\frac{1}{\lambda}\frac{\partial}{\partial\alpha}\left(\mathrm{Li}_{\alpha+1}(-e^{-\lambda})-\mathrm{Li}_{\alpha+1}(-e^{\lambda})\right)\Big|_{\alpha=0}\right], \tag{28}$$

$$f_2(\lambda) \;=\; \left|\Gamma\left(\tfrac{8}{3}\right)\left(\mathrm{Li}_{5/3}(-e^{-\lambda})-\mathrm{Li}_{5/3}(-e^{\lambda})\right)\right|^{3/5}, \tag{29}$$

$$f_3(\lambda) \;=\; \left|\Gamma\left(\tfrac{10}{3}\right)\left(\mathrm{Li}_{7/3}(-e^{-\lambda})-\mathrm{Li}_{7/3}(-e^{\lambda})\right)\right|^{3/7}, \tag{30}$$

$$f_4(\lambda) \;=\; \left|\lambda(\lambda^2+\pi^2)\right|^{1/3}. \tag{31}$$

We note that the dependence on q^* indeed drops out in the sum of the transverse and longitudinal parts.

In the zero temperature limit ($|\lambda|\to\infty$) we have $f_i(\lambda)\to|\lambda|$. If the temperature is much higher than $|E-\mu|$ (i.e. $\lambda\to 0$) we have $f_1(\lambda)\to c_0 := \frac{\pi}{2}\exp(1-\gamma_E)=2.397357\ldots$ and $f_{2,3,4}(\lambda)\to 0$. For $|\lambda|\gg c_0$ or $|\lambda|\ll c_0$ we may approximate $f_1(\lambda)$ with $\max(c_0,|\lambda|)$, which is qualitatively the result quoted in [6]. It should be noted, however, that the calculation of Ref. [6] only took into account transverse gauge bosons, and therefore the scale under the logarithm and its parametric dependence on the coupling was not correctly rendered.

For the imaginary part we find

$$\begin{aligned}\mathrm{Im}\,\Sigma_+ - \mathrm{Im}\,\Sigma_+|_{E=\mu} = g^2C_f m\Bigg[&-\frac{T}{24\pi m}g_1\left(\frac{\varepsilon}{T}\right)+3\frac{2^{1/3}}{45\pi^{7/3}}\left(\frac{T}{m}g_2\left(\frac{\varepsilon}{T}\right)\right)^{5/3}\\ &-\frac{1}{64\sqrt{2}}\left(\frac{T}{m}g_3\left(\frac{\varepsilon}{T}\right)\right)^2+20\frac{2^{2/3}}{63\pi^{11/3}}\left(\frac{T}{m}g_4\left(\frac{\varepsilon}{T}\right)\right)^{7/3}+\mathcal{O}\left(\left(\frac{T}{m}\right)^3\right)\Bigg],\end{aligned} \tag{32}$$

where

$$g_1(\lambda) \;=\; 2\log\cosh\left(\frac{\lambda}{2}\right), \tag{33}$$

$$g_2(\lambda) \;=\; \left[-\Gamma\left(\tfrac{8}{3}\right)\left(\mathrm{Li}_{5/3}(-e^{-\lambda})+\mathrm{Li}_{5/3}(-e^{\lambda})+2\left(1-2^{-2/3}\right)\zeta\left(\tfrac{5}{3}\right)\right)\right]^{3/5}, \tag{34}$$

$$g_3(\lambda) \;=\; |\lambda|, \tag{35}$$

$$g_4(\lambda) \;=\; \left[-\Gamma\left(\tfrac{10}{3}\right)\left(\mathrm{Li}_{7/3}(-e^{-\lambda})+\mathrm{Li}_{7/3}(-e^{\lambda})+2\left(1-2^{-4/3}\right)\zeta\left(\tfrac{7}{3}\right)\right)\right]^{3/7}. \tag{36}$$

In the zero temperature limit we have $g_i(\lambda)\to|\lambda|$. If the temperature is much higher than $|E-\mu|$ we have $g_i(\lambda)\to 0$.

Explicitly, our $T=0$ result reads

$$\begin{aligned}\Sigma_+|_{T=0} \;=\; \frac{M_\infty^2}{2E}-g^2C_f m\Bigg\{&\frac{\varepsilon}{12\pi^2 m}\left[\log\left(\frac{4\sqrt{2}m}{\pi|\varepsilon|}\right)+1\right]+\frac{i|\varepsilon|}{24\pi m}\\ &+\frac{2^{1/3}\sqrt{3}}{45\pi^{7/3}}\left(\frac{|\varepsilon|}{m}\right)^{5/3}(\mathrm{sgn}(\varepsilon)-\sqrt{3}i)\\ &+\frac{i}{64\sqrt{2}}\left(\frac{\varepsilon}{m}\right)^2-20\frac{2^{2/3}\sqrt{3}}{189\pi^{11/3}}\left(\frac{|\varepsilon|}{m}\right)^{7/3}(\mathrm{sgn}(\varepsilon)+\sqrt{3}i)\\ &-\frac{6144-256\pi^2+36\pi^4-9\pi^6}{864\pi^6}\left(\frac{\varepsilon}{m}\right)^3\left[\log\left(\frac{0.928\,m}{|\varepsilon|}\right)-\frac{i\pi\mathrm{sgn}(\varepsilon)}{2}\right]\\ &+\mathcal{O}\left(\left(\frac{|\varepsilon|}{m}\right)^{11/3}\right)\Bigg\},\end{aligned} \tag{37}$$

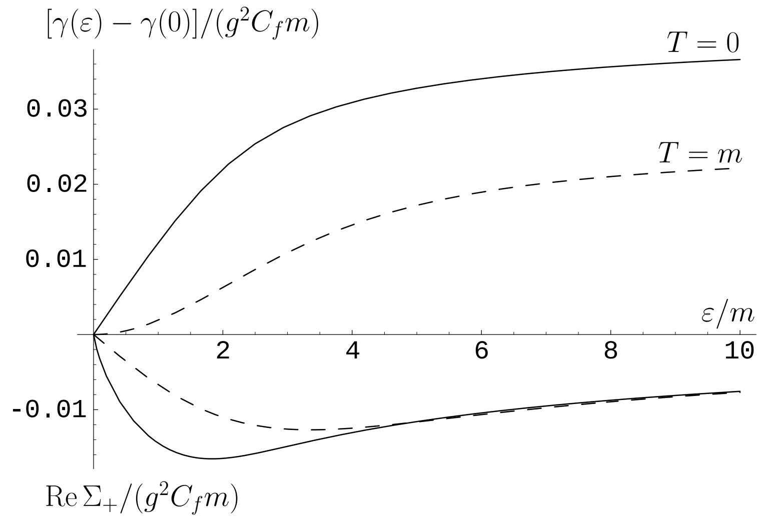

FIG. 1: Real and imaginary part of $\Sigma_+^{(s)}/(g^2C_fm)$ as a function of $\varepsilon/m \equiv (E-\mu)/m$ at $T=0$ (full lines) and $T=m$ (dashed lines).

where the scale of the last logarithm was determined by resumming IR enhanced contributions [22].

Apart from the first logarithmic term, the leading imaginary parts contributed by the transverse and longitudinal gauge bosons were known previously [19, 23, 24]. As our results show, the damping rate obtained by adding these two leading terms [19, 23] is actually incomplete beyond the leading term, because the subleading transverse term of order $|\varepsilon|^{5/3}$ is larger than the leading contribution from Σ_L.

IV. NUMERICAL RESULTS AND DISCUSSION

At large values of $E-\mu$ or at large negative values of E, where one obtains the self energy of the antiquasiparticles, the soft contribution $\mathrm{Re}\,\Sigma^s$ can be shown to vanish. At $T=0$, where the imaginary part does not contain an infrared divergent contribution, $\mathrm{Im}\,\Sigma_+$ approaches the constant[1]

$$\lim_{E\to\infty} \mathrm{Im}\,\Sigma_+(E)\Big|_{T=0} = -g^2C_fm \times 0.040534\ldots \tag{38}$$

The resulting damping constant $\gamma = -\mathrm{Im}\,\Sigma_+$ is also that of the antiquasiparticles, which are of course far from their nonexistent Fermi surface for $\mu > 0$.

At intermediate energies $|E-\mu| \gtrsim m$, both the real and imaginary parts of Σ_+ are nontrivial functions that we have evaluated numerically. The results are shown in Fig. 1 for the two cases $T=0$ and $T=m$. Since at finite temperature the imaginary part of Σ contains a (constant) infrared singular contribution, we plot $\gamma(E-\mu)-\gamma(0)$ instead. The latter function is even with respect to its argument, resulting in a cusp at $E=\mu$ for $T=0$, while at finite T the damping contribution vanishes quadratically at $E=\mu$. The real part is an odd function with respect to $E-\mu$. Again, $E=\mu$ is a nonanalytic point at $T=0$, but analytic at finite T.

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[1] The numerical constant in Eq. (38) agrees with the value given with two significant digits in Ref. [24] (taking into account the different normalization), and it is very close to, but not in complete agreement with, the value $(\frac{1}{24\pi}+\frac{1}{64})\sqrt{2} \approx 0.040854$ quoted in Ref. [19].

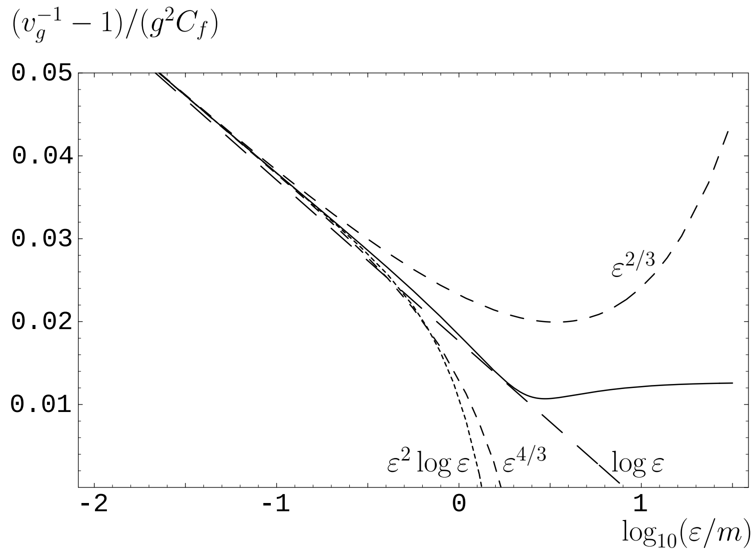

FIG. 2: $(v_g^{-1}-1)/(g^2C_f)$ as a function of $\log_{10}((E-\mu)/m)$ at zero temperature.

At $T=0$, the consequence of the nonanalyticity at $E=\mu$ is that the group velocity dE/dp together with the residue in the propagator vanishes. The group velocity is determined by

$$v_g^{-1}=1+\frac{M_\infty^2}{2E^2}-\frac{\partial \mathrm{Re}\,\Sigma_+^{(\mathrm{s})}}{\partial E}. \tag{39}$$

The numerical result for $v_g^{-1}-1$ is given in Fig. 2 together with the series expansion for small $|E-\mu|$ following from the results of the previous section. As one can see, this expansion converges well only for $|E-\mu|\ll m$, and it turns out that the logarithmic contribution

$$v_g^{-1}(E-\mu)=1+\frac{g^2C_f}{12\pi^2}\left[\ln\frac{4\sqrt{2}\,m}{\pi|E-\mu|}+\frac{3}{2}\right]+O\Big(\Big(\frac{E-\mu}{m}\Big)^{2/3}\Big) \tag{40}$$

is already a rather good approximation up to the point where one should switch to the leading order result $M_\infty^2/(2E^2)=g^2C_f/(8\pi^2)\approx 0.013g^2C_f$ that is relevant for larger values of $m\lesssim|E-\mu|\ll\mu$.

At small finite temperature the results of the previous section show that the growth of v_g^{-1} for $E\to\mu$ is limited by

$$v_g^{-1}=1+\frac{g^2C_f}{12\pi^2}\ln\frac{c_0'm}{T}+O\Big(\Big(\frac{T}{m}\Big)^3\Big) \tag{41}$$

with $c_0'=4\sqrt{2}e^{5/2}/(\pi c_0)\approx 9.15016$. In this case the group velocity at and above the Fermi surface can be approximated by

$$v_g^{-1}(E-\mu)\approx 1+\frac{g^2C_f}{12\pi^2}\max\left\{\min\left(\ln\frac{9.15\,m}{T},\ln\frac{8.07\,m}{|E-\mu|}\right),\frac{3}{2}\right\}. \tag{42}$$

This result for the group velocity of the quasiparticle excitations is for example of direct relevance for the calculation of the neutrino emission from normal degenerate quark matter [25] which is enhanced by non-Fermi-liquid effects [13]. As was shown recently in Ref. [13], the neutrino emissivity involves two powers of $\alpha_s\ln(m/T)$ from the quasiparticle group velocities, which overcompensate the single power of $\alpha_s\ln(m/T)$ in the specific heat that counteracts in the cooling

rate. For a numerical evaluation one evidently needs to know the constants under these logarithms. Interestingly enough, the constants under the log's in (42) are much larger than the constant under the log arising in the specific heat determined in Ref. [11, 12] as $\log(0.282m/T)$ — for more discussion see Ref. [22].

V. CONCLUSIONS

In this article we have computed the fermion self energy in an ultradegenerate relativistic plasma. For small $|E-\mu|$ and small T we have obtained a perturbative expansion of Σ_+ beyond the leading logarithm that is responsible for non-Fermi-liquid behavior. We found that dynamical screening leads to fractional powers in this series, which are analogous to the fractional powers in the anomalous specific heat in normal degenerate quark matter [11, 12]. Furthermore we have performed a numerical computation of the self energy and the group velocity for larger values of $|E-\mu|$. Our results provide an important ingredient for quantitative calculations of non-Fermi-liquid effects such as the computation of the enhanced neutrino emissivity of ungapped quark matter [13].

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